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Pure Braids and Group Actions from 3-fold Flops

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Abstract

The first part of this thesis gives a new, simplified presentation of the classical pure braid group. The generators are given by the squares of the longest elements over connected subgraphs, and we prove that the only relations are either commutators or certain palindromic length 5 box relations. This presentation is motivated by twist functors in algebraic geometry, but the proof is entirely Coxeter-theoretic. We also prove that the analogous set does not generate for all Coxeter arrangements, which in particular answers a question of Donovan and Wemyss.

The second part of this thesis calculates restricted roots associated to an ADE Dynkin graph Δ together with a choice $\mathcal{J}$ of vertices in Δ, with $|\Delta| - |\mathcal{J}| = 3$. The restricted roots correspond to hyperplanes in the intersection arrangement inside $\mathbb{R}^3$. We then classify these intersection arrangements via mutation classes for each $\mathcal{J}$, and there are precisely 10 hyperplane arrangements inside $\mathbb{R}^3$. In particular, this part of the thesis extends [Iyama and Wemyss, 2020] by giving finite classifications of hyperplanes when $|\Delta| - |\mathcal{J}| > 2$.

The last part of this thesis constructs derived autoequivalences associated to an algebraic flopping contraction $X \to X_{\mathrm{con}}$, where X is quasi-projective with only mild singularities. These functors are constructed naturally using bimodule cones, and we prove these cones are locally two-sided tilting complexes by using local-global properties and a key commutative diagram. The main result is that these autoequivalences combine to give an action of the fundamental group of an associated infinite hyperplane arrangement on the derived category of X. This generalises and simplifies [Donovan and Wemyss, 2019], by finally removing reliance on subgroups, and it also lifts many other results from the complete local setting.

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