Computing the Artin Component using Reconstruction Algebras

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ABSTRACT

This thesis studies noncommutative resolutions of non-Gorenstein singularities and uses them to construct classical deformation spaces. In the first part, it recovers the Artin component of the deformation space of a cyclic surface singularity using only the quiver of the corresponding reconstruction algebra. The relations of the reconstruction algebra are then deformed, and the deformed relations together with variation of the Geometric Invariant Theory (GIT) quotient achieve the simultaneous resolution. This extends work of Brieskorn, Kronheimer, Grothendieck, Cassens–Slodowy and Crawley-Boevey–Holland into the setting of singularities C2/H with $H \leq GL(2, C)$, and furthermore gives a prediction for what is true more generally. Additionally, outside the toric setting, the thesis demonstrates the construction of simultaneous resolution for determinantal surfaces, which are a specific type of rational surface singularities. The main new di↵erence to the above case is that, in addition to the quiver of the reconstruction algebra, certain noncommutative relations, namely those of the canonical algebra of Ringel, are required. All the relations of the reconstruction algebra except the canonical relation are then deformed, and these deformed relations together with variation of the GIT quotient achieve the simultaneous resolution.